

CS103, AUTUMN 2026



Lecture 08:

Set Theory Revisited

Set Theory Revisited

1. Announcements
2. Quick Recap
3. How Not to Prove Things About Sets
4. Definitions (and the Assume-Prove Table for Sets)
5. Proofs on Subsets
6. Unions and Intersections
7. More Announcements
8. Set Equality
9. Set-Builder Notation
10. Power Set Membership
11. Action Items
12. What's Next?
13. Appendix: Additional Set Proof Examples

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Problem Set Submissions

- ***Tagging partners on coding portion.***
 - Contrary to what I said on Wednesday, you can jointly submit both the coded *and* written portions of problem sets. Sorry about that!
- ***Please use LaTeX Template (or its structure).***
 - Please check out the LaTeX templates we provide with each problem set and follow the provided structure (within reason). For example, if the template has a table to fill in, it is enormously helpful to the TAs if everyone submits responses in tabular format.
- ***Pleas tag problems.***
 - Tagging each individual problem is super helpful to TAs and does not change your submission timestamp. Untagged problems might incur grading penalties and/or receive no credit.

Deadlines and Goings-On

- ***Sunday, Oct. 11, 11:59 PM***
 - Finalize recitation opt-in / opt-out.
- ***Next Monday through Friday, Oct. 16, at 11:59 PM***
 - Lecture attendance (PollEV) opt-out form.
- ***Monday, Oct. 12, 11:59 PM***
 - Left-handed exam desk form.
- ***Wednesday, Oct. 14, 11:59 PM***
 - Problem Set 1 regrade requests due.
- ***Friday, Oct. 16, 1:00 PM***
 - Problem Set 3 is due. The problem set is now posted!

Recitation Things

- ***Dashboard***

- The recitation dashboard continues to evolve and now shows your attendance record.

- ***Recitation Problem Solutions***

- These are posted every Wednesday, sometime in the evening, after the last recitation has concluded. Be sure to look over these, even if you've opted out of recitation. There are a lot of important gems there, and many of our problems are informed by common errors and misunderstandings that we see, which you will want to clear up before the midterm!

- ***Attendance Credit***

- Please be sure to arrive to recitation on time.

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The Assume-Prove Table

	If you assume this is true...	To prove that this is true...
$\forall x. A$	Initially, do nothing . Once you find a z through other means, you can state it has property A .	Have the reader pick an arbitrary x . We then prove A is true for that choice of x .
$\exists x. A$	Introduce a variable x into your proof that has property A .	Find an x where A is true. Then prove A is true for that specific choice of x .
$A \rightarrow B$	Initially, do nothing . Once you know A is true, you can conclude B is also true.	Assume A is true, then prove B is true.
$A \wedge B$	Assume A . Also assume B .	Prove A . Also prove B .
$A \vee B$	Consider two cases. Case 1: A is true. Case 2: B is true.	Either prove $\neg A \rightarrow B$ or prove $\neg B \rightarrow A$. <i>(Why does this work?)</i>
$A \leftrightarrow B$	Assume $A \rightarrow B$ and $B \rightarrow A$.	Prove $A \rightarrow B$ and $B \rightarrow A$.
$\neg A$	Simplify the negation, then consult this table on the result.	Simplify the negation, then consult this table on the result.

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Claim: If A , B , and C are sets where $A \in B$ and $B \in C$, then $A \in C$.

Proof (?): Assume A , B , and C are sets where $A \in B$ and $B \in C$.

We need to show that $A \in C$.

Since $A \in B$, we know that A is contained in B . Since $B \in C$, we know that B is contained in C . Therefore, because A is contained in B and B is contained in C , we know that A is contained in C . This means that $A \in C$, as required. ■

Claim: If A , B , and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

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Which of these claims are true (if any)?

Answer at

cs103.stanford.edu/pollev

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This claim is not true! For example...

$$\begin{aligned}\emptyset &\in \{\emptyset\} \\ \{\emptyset\} &\in \{\{\emptyset\}\} \\ \emptyset &\notin \{\{\emptyset\}\}\end{aligned}$$

Lie: If A , B , and C are sets where $A \in B$ and $B \in C$, then $A \in C$.

Bad Proof: Assume A , B , and C are sets where $A \in B$ and $B \in C$.
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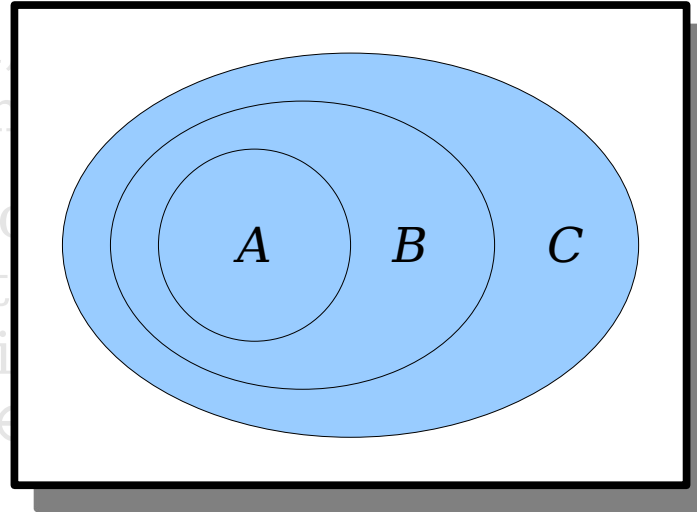
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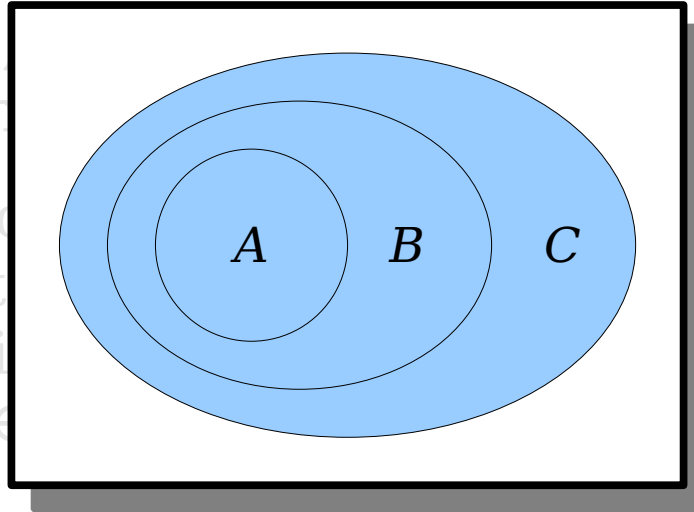
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This can't be a good proof;
the same basic argument
proves a false claim!

Claim: Let A , B , and C be sets. If $A \subseteq B$ and $A \subseteq C$, then we have $A \subseteq B \cap C$.

Proof (?): Assume $A \subseteq B$ and $A \subseteq C$. We need to show $A \subseteq B \cap C$.

Since $A \subseteq B$, all elements of A are in B . Since $A \subseteq C$, all elements of A are also in C . Therefore, all elements of A are in both B and C . Therefore, we see that $A \subseteq B \cap C$. ■

Claim: Let A , B , and C be sets. If $A \subsetneq B$ and $A \subsetneq C$, then we have $A \subsetneq B \cap C$.

Proof (?): Assume $A \subsetneq B$ and $A \subsetneq C$. We need to show $A \subsetneq B \cap C$.

Since $A \subsetneq B$, all elements of A are in B and there are other elements of B . Since $A \subsetneq C$, all elements of A are also in C and there are other elements of C . Therefore, all elements of A are in both B and C , and there are some other elements in B and C . Therefore, we see that $A \subsetneq B \cap C$. ■

(Reminder: $S \subsetneq T$ means
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Which of these claims are true (if any)?

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This claim is not true! For example...

$$\{137\} \subsetneq \{137, 103\}$$

$$\{137\} \subsetneq \{137, 107\}$$

$$\text{But } \{137\} = \{137, 103\} \cap \{137, 107\}$$

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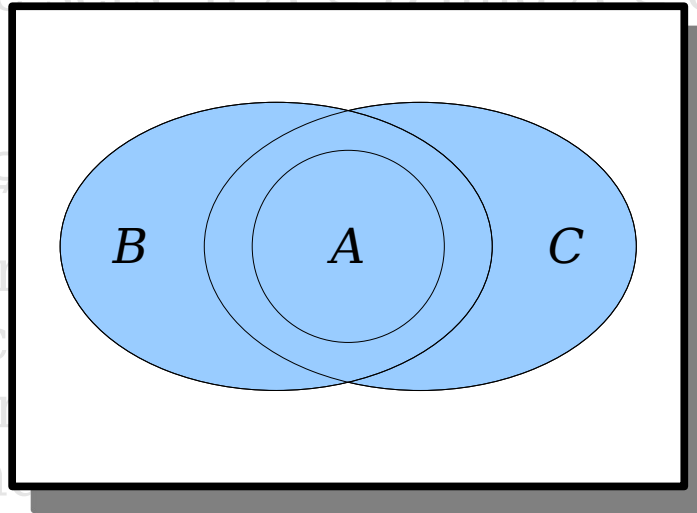
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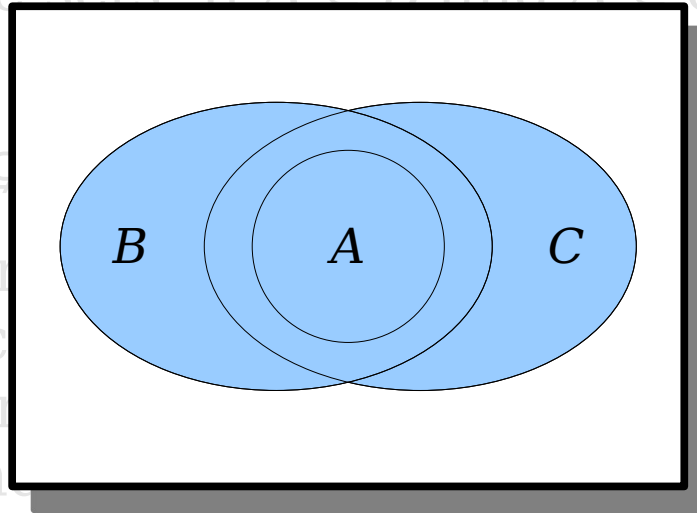
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What Went Wrong?

- The style of arguments you've just seen are **not** how to prove results on sets.
- Here are two particular things that went wrong:
 - The reliance on high-level terms like “contained” is not mathematically precise.
 - A discussion of “all elements” of a set is not how to reason about collections of objects.
- **Question:** How do we write rigorous proofs about sets?

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The Importance of Definitions

- As you've seen in recent lectures, formal definitions underpin mathematical proofs.
- The major issue from the previous proofs is that we haven't defined what our terms mean.
 - How do we define what $A \in B$ means?
 - How do we define what $A \subseteq B$ means?
 - How do we define what $A \cap B$ means?
- Think back to our proof writing triad: we currently have intuitions for these concepts, but not formal definitions.

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$			
$S = T$			
$x \in S \cap T$			
$x \in S \cup T$			
$X \in \wp(S)$			
$x \in \{y \mid P(y)\}$			

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Defining Subsets

- Formally speaking, if S and T are sets, we say that $S \subseteq T$ when the following holds:

$$\forall x \in S. x \in T$$

- Now, suppose you're working with a proof where you encounter $S \subseteq T$. Think back to the proof table.
 - To **assume** that $S \subseteq T$, what should you do?
 - To **prove** that $S \subseteq T$, what should you do?

Answer at

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	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$			
$x \in S \cap T$			
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A Correct Proof on Sets

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Proof:

A Correct Proof on Sets

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Since $x \in A$ and $A \subseteq B$, we know $x \in B$. Similarly, since $x \in B$ and $B \subseteq C$, we see that $x \in C$, as required.

	Is defined as...	If you assume this is true...	To prove that this is true...
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Let's compare this to our previous approach.

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Bad Proof: Assume A , B , and C are sets where $A \subseteq B$ and $B \subseteq C$. We need to show that $A \subseteq C$.

Since $A \subseteq B$, we know that A is contained in B . Since $B \subseteq C$, we know that B is contained in C . Therefore, because A is contained in B and B is contained in C , we know that A is contained in C . This means that $A \subseteq C$, as required. ■

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Theorem: Let A , B , and C be sets. If $A \subseteq B$
and $A \subseteq C$, then $A \subseteq B \cap C$.

Unions and Intersections

- The statement $x \in S \cap T$ is defined as

$$x \in S \quad \wedge \quad x \in T.$$

- The statement $x \in S \cup T$ is defined as

$$x \in S \quad \vee \quad x \in T.$$

- These are operational definitions: they show how unions and intersections interact with the $\in$ relation rather than saying what the union or intersection of two sets “are.”

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$			
$x \in S \cap T$	$x \in S \wedge x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \vee x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
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Bad Proof: Assume $A \subseteq B$ and $A \subseteq C$. We need to show $A \subseteq B \cap C$.

Since $A \subseteq B$, all elements of A are in B . Since $A \subseteq C$, all elements of A are also in C . Therefore, all elements of A are in both B and C . Therefore, we see that $A \subseteq B \cap C$. ■

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Midterm Exam Logistics

- Our first midterm exam is **Monday, October 19th**, from **7-10 PM**. Locations vary.
- You're responsible for Lectures 00 – 08 and topics covered in PS1 – PS3. This includes the assume-prove table. As far as functions go, if you're solid on the first-order logic definitions and the application of the assume-prove table to function proofs, you should be fairly set for how functions will appear on the exam. Exam problems may build on the written or coding components from the problem sets.
- The exam is closed-book, closed-computer, and limited-note. You can bring a double-sided, 8.5" × 11" sheet of notes with you to the exam, decorated however you'd like.

Midterm Exam Logistics

- Students with alternate exam arrangements: you should have heard from us with details. If you haven't, contact Joyce and Sean ASAP.
- We have posted an ***Exam Logistics*** page on the course website with full details and logistics. We will add a few details and practice problems this weekend.
- The page also includes advice from former CS103 students about how to do well on the exams.
- Check it out. There are tons of goodies there to help you prepare for the exam.

Exam Day Logistics

- We'll have proctors in the room.
- We have assigned seating; see course website. These will be posted sometime next Tuesday, Oct. 13.
- Check out your seat in advance, and screenshot it!
- No phones, calculators, or other digital devices during the exam.
- No belongings near your seats (leave them in your dorms or at the front of the classroom).
- Must have Stanford student ID to turn in exam.
- We will at some point take a picture of the exam room.

Extra Practice Problems

- Later today, you'll find Extra Practice Problems 1 on the website: a collection of practice midterms and an assortment of other questions.
- ***Our Recommendation:***
 - Work through one or two practice exams under ***realistic conditions*** (block off three hours, have your notes sheet, use pencil and paper).
 - Review the solutions only when you're done. ***Don't peek!*** You can't do that on the actual exam.
 - Ping the course staff to ask questions, whether that's "please review this proof I wrote for one of the exam questions" or "why doesn't the solution do X, which seems easier than Y, which is what it did?"
 - ***Internalize the feedback.*** What areas do you need more practice with? Study up on those topics. What transferrable skills did you learn in the course of solving the problems? If you aren't sure, ask!
 - Repeat!
- Realistically, we don't expect you to do all of the practice exams. We've provided those just so you can get a sense of what's out there.

Review Session

- One or more of our amazing TAs will be holding a review session sometime next week – either Thursday or Friday. Date, time, and location will be posted on Ed!
- Come prepared to discuss any questions you may have.
- You'll get more out of this session if you have done some preliminary study first.
- We will post a recording of the session for those who can't attend synchronously, but robust attendance helps us better serve you.

Final Thoughts

- We want everyone to do well on the exam!
- Let us know if you have any questions as we approach the exam – whether logistical in nature or content-related.

Also!

- One guide and one checklist unlocking today.
- Be mindful of all the deadlines listed at the top

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Theorem: Let A and B be sets. Then
 $A \subseteq B$ if and only if $A \cup B = B$.

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$	$S \subseteq T \wedge T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in S \cap T$	$x \in S \wedge x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \vee x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
$x \in \{y \mid P(y)\}$			

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Proof:

See appendix!

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Set-Builder Notation

- Let S be the set defined here:

$$S = \{ n \mid n \in \mathbb{N} \text{ and } n \geq 137 \}$$

- Now imagine you have some quantity x . Based on this...
 - ... if you **assume** that $x \in S$, what does that tell you about x ?
 - ... if you need to **prove** that $x \in S$, what do you need to prove?

Answer at

cs103.stanford.edu/pollev

Set-Builder Notation

- Like unions and intersections, we have an operational definition for set-builder notation. It's the following:

Let $S = \{ y \mid P(y) \}$.

Then $x \in S$ when $P(x)$ is true.

- So, for example:
 - $x \in \{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \}$ means $x \in \mathbb{N}$ and x is even.
 - $x \in \{ n \mid \exists k \in \mathbb{N}. n = 2k + 1 \}$ means that there is a $k \in \mathbb{N}$ where $x = 2k + 1$. (Equivalently, x is an odd natural number)
- **Key Point:** The placeholder variable disappears in all these examples. After all, *it's just a placeholder*.

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$	$S \subseteq T \wedge T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
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$x \in S \cup T$	$x \in S \vee x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
$x \in \{y \mid P(y)\}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Proofs on Set-Builder Notation

Some Useful Notation

- If n is a natural number, we define the set **$[n]$** as follows:

$$[n] = \{ k \mid k \in \mathbb{N} \wedge k < n \}$$

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- If n is a natural number, we define the set **$[n]$** as follows:

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 - $[3] =$

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- So, for example:
 - $[3] = \{0, 1, 2\}$
 - $[0] =$

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- So, for example:
 - $[3] = \{0, 1, 2\}$
 - $[0] = \emptyset$
 - $[5] = \{0, 1, 2, 3, 4\}$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$,
then $[m] \subseteq [n]$

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What We're Assuming

$$m \in \mathbb{N}$$

$$n \in \mathbb{N}$$

$$m < n$$

$$[z] = \{ k \mid k \in \mathbb{N} \wedge k < z \}$$

What We Need to Prove

$$[m] \subseteq [n]$$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

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Since $x \in [m]$, we know that $x \in \mathbb{N}$ and $x < m$. Then, because $x < m$ and $m < n$, we know that $x < n$. Collectively this means that $x \in \mathbb{N}$ and $x < n$, so $x \in [n]$, as required.

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Notice that *there is no set-builder notation in this proof*. We were able to avoid it by using the rules for what $x \in \{y \mid P(y)\}$ say to do.

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Your Action Items

- ***Read “Guide to Proofs on Discrete Structures.”***
 - There’s additional guidance and practice on the assume/prove dichotomy and how it manifests in problem-solving.
- ***Read “Discrete Structures Proofwriting Checklist.”***
 - Keep the items here in mind when writing proofs. We’ll use this when grading your problem set.
- ***Read “Guide to Proofs on Sets.”***
 - There’s some good worked examples in there to supplement today’s lecture, several of which will be relevant for the problem set.
- ***Start Problem Set 3.***
 - Start early and make slow and steady progress.
- ***(Optional) Look over Extra Practice Problems 1***
 - For extra pre-exam review. These will be posted later today.
- ***Various Deadlines***
 - See the top of today’s slide deck for various other important deadlines.

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Next Time

- ***Graph Theory***
 - A ubiquitous, powerful abstraction with applications throughout computer science.
- ***Vertex Covers***
 - Making sure tourists don't get lost.
- ***Independent Sets***
 - Helping the recovery of the California Condor.

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Appendix:
More Sample Set Proofs!

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

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What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

What I Need to Show

$$A \cup B \subseteq C \cup D$$

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

What I Need to Show

$$\cancel{A \cup B \subseteq C \cup D}$$

$$\forall x \in A \cup B. x \in C \cup D$$

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

$$x \in A \cup B$$

What I Need to Show

~~$$A \cup B \subseteq C \cup D$$~~

~~$$\forall x \in A \cup B. x \in C \cup D$$~~

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

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$$\forall z \in B. z \in D$$

$$x \in A \cup B$$

What I Need to Show

~~$$A \cup B \subseteq C \cup D$$~~

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$$x \in C \text{ or } x \in D$$

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

$$x \in A \cup B$$

$$\text{Case 1: } x \in A$$

$$\text{Case 2: } x \in B$$

What I Need to Show

~~$$A \cup B \subseteq C \cup D$$~~

~~$$\forall x \in A \cup B. x \in C \cup D$$~~

$$x \in C \text{ or } x \in D$$

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Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

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Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$.

Theorem: Let A , B , C , and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

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Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

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Case 1: $x \in A$.

Case 2: $x \in B$.

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Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

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In either case, we see that $x \in C \cup D$, as required.

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Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$.

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Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$.

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Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

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($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$. And since $x \in A \cup B$ and $A \cup B = B$, we see that $x \in B$, as required.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

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Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$. And since $x \in A \cup B$ and $A \cup B = B$, we see that $x \in B$, as required. ■

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

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if $\wp(A) = \wp(B)$, then $A = B$.

<i>What I'm Assuming</i>	<i>What I Need to Show</i>
$\wp(A) = \wp(B)$ $\wp(A) \subseteq \wp(B)$ $\forall Z \in \wp(A). Z \in \wp(B)$ $\wp(B) \subseteq \wp(A)$ $\forall Z \in \wp(B). Z \in \wp(A)$	$A = B$

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$$\wp(A) = \wp(B)$$

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What I Need to Show

$$\cancel{A = B}$$

$$A \subseteq B$$

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Theorem: Let A and B be sets. Then
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What I Need to Show

$$\cancel{A = B}$$

$$\cancel{A \subseteq B}$$

$$\forall x \in A. x \in B.$$

$$\cancel{B \subseteq A}$$

$$\forall z \in B. z \in A.$$

Theorem: Let A and B be sets. Then
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<i>What I'm Assuming</i>	<i>What I Need to Show</i>
$\wp(A) = \wp(B)$	$A = B$
$\wp(A) \subseteq \wp(B)$	$A \subseteq B$
$\forall Z \in \wp(A). Z \in \wp(B)$	$\forall x \in A. x \in B.$
$\wp(B) \subseteq \wp(A)$	$B \subseteq A$
$\forall Z \in \wp(B). Z \in \wp(A)$	$\forall z \in B. z \in A.$
$x \in A$	
$\{x\} \subseteq A$	
$\{x\} \in \wp(A)$	

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$\wp(A) = \wp(B)$	$A = B$
$\wp(A) \subseteq \wp(B)$	$A \subseteq B$
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$\wp(B) \subseteq \wp(A)$	$B \subseteq A$
$\forall Z \in \wp(B). Z \in \wp(A)$	$\forall z \in B. z \in A.$
$x \in A$	$x \in B$
$\{x\} \subseteq A$	$\{x\} \subseteq B$
$\{x\} \in \wp(A)$	$\{x\} \in \wp(B)$

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Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

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Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$.

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Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$.

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Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$. Thus we see that $\{x\} \subseteq B$, which in turn means that $x \in B$, as required. ■